Exercise 24

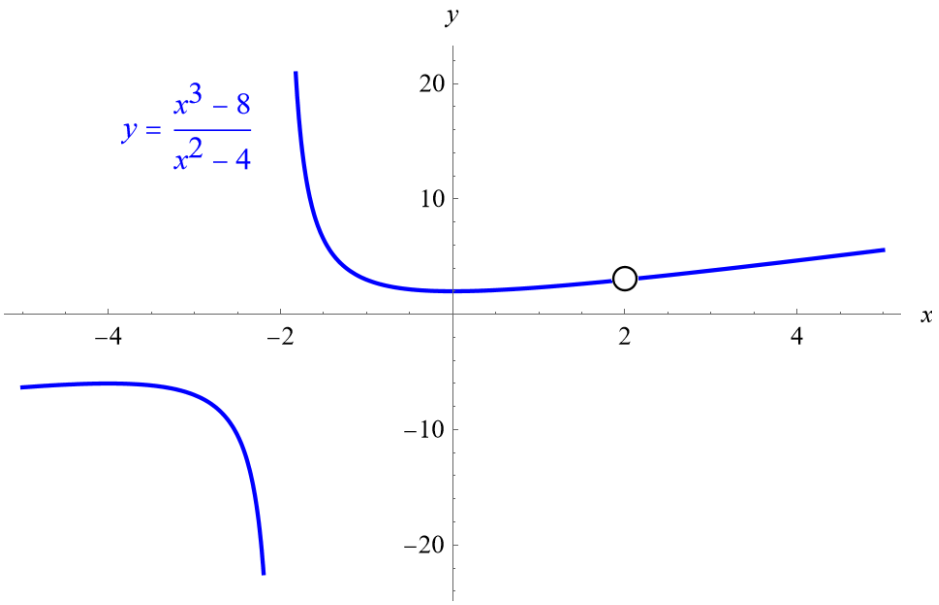
How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

$$f(x) = \frac{x^3 - 8}{x^2 - 4}$$

Solution

Notice that because the factor of $x - 2$ cancels out in the denominator, a hole (removable discontinuity) is left in the graph at $x = 2$.

$$\begin{aligned} f(x) &= \frac{x^3 - 8}{x^2 - 4} \\ &= \frac{(x-2)(x^2 + 2x + 4)}{(x+2)(x-2)} \\ &= \frac{x^2 + 2x + 4}{x + 2} \end{aligned}$$



Remove the discontinuity by defining $f(2) = \frac{2^2+2(2)+4}{2+2} = 3$.

$$f(x) = \begin{cases} \dfrac{x^3 - 8}{x^2 - 4} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$$